## Artefact 2 addresses the following focus areas:

- 2.1 Demonstrate knowledge and understanding of the concepts, substance and structure of the content and teaching strategies of the teaching area (content highlighted in blue)
- 2.2 Organise content into an effective learning and teaching sequence (content highlighted in green)
- 2.3 Use curriculum, assessment and reporting knowledge to design learning sequences and lesson plans (content highlighted in yellow)
- 2.5 Know and understand literacy and numeracy teaching strategies and their application in teaching areas (content highlighted in blue)
- 2.6 Information and Communication Technology (ICT) (content highlighted in red)

## EDMA310/360 Mathematics unit planner

{Jessica Vella – S00144421}

**Unit Overview** 

Unit title: Understanding Patterns

**Content maths area:** Algebra

Grade/year level: Level 3

Learning Focus (ideas extrapolated from AusVELS): Strand: Number and Algebra Sub strand: Patterns and Algebra

Describe, continue, and create number patterns resulting from performing addition or subtraction (ACMNA060)

## Rationale:

Algebra is an important mathematical content area that is explored in primary schools, as students are able to study patterns and relationships (Reys et al, 2013). Algebra is a way of thinking; algebra provides students with strategies for analysing representations, generalising and justifying (Reys et al, 2013; Warren and Cooper, 2007 and Wilkie 2014). Those who are involved in algebraic thinking have a better understanding of the mathematical processes,

structures and properties of mathematics (Hunter, 2015; Reys et al, 2013; Stacey & MacGregor, 1997 and Warren, Mollinson & Oestrich, 2009). Patterns are an area that is covered in algebra that is very important. Creating, constructing and describing patterns require problem-solving skills. Patterns help students develop number sense, ordering, counting and sequencing (Reys et al, 2013). It helps develop thinking strategies as well as algebraic thinking.

## Assumed prior knowledge of students:

Students have had many previous experiences with patterns such as copying, continuing patterns using material and drawings, students have had experiences with describing number patterns formed by skip counting and describing patterns with numbers. Students have also been able to identify missing elements in patterns (AusVELS, 2015).

## Grouping strategies to support learning:

The grouping strategies that will be used in this unit are mixed ability grouping and pair work. Students will also have an opportunity to work on their own, so they are able to think independently on the maths involved in the task and demonstrate their own mathematical strategies. Allowing students to work with other students in the classroom promotes social interactions and collaborative learning (Hoffman, 2010). Every student is unique and has their own talents and strengths, therefore students are able to learn from one another (Sullivan, 1997). It is very beneficial when students work with others as it gives them an opportunity to have discussions, explain their thinking, clarify any misconceptions, share ideas and most importantly learn from one another (Hoffman, 2010 and Sullivan, 1997). Giving students an opportunity to work with others is found to be the most effective way to meet the instructional needs of students and allow collaborative opportunities to occur (Hoffman, 2010). Reys et al (2013), states that giving students opportunities for discussion, collaboration, sharing and mutual encouragement amongst students plays an important role in helping students become skilful problem solvers.

## Overview of assessment:

The assessment strategies that will be used in this unit are formative and affective assessment strategies. Formative assessment is assessment for and as learning that impacts on future teaching and learning opportunities, whereas affective assessment is learning associated with interests, motivations, confidence, attitudes, and values (Reys et al, 2013). Assessment will take place through observation, the teacher will observe students' performance, strengths and weaknesses. They will identify what strategies they used, if they needed equipment to assist them with their learning and listen when they are explaining their mathematical thinking. The methods that will be used to record assessment information will be through anecdotal notes, checklists and rubrics (Reys et al, 2013).

Open tasks will also be given to students this unit so teachers can assess how students approach the task, what their mathematical knowledge and thinking is and identify what strategies they used (Varygiannes, 2014). Open tasks are effective as they enable the whole class to participate, they can easily be modified or extended in order to scaffold learning (Ferguson, 2009). Varygiannes (2014) states that open tasks promote "engagement in tasks that will enable our students to reason effectively, use systems thinking, make judgments and decisions, and solve problems" (p.278). Students will also be given an opportunity after each maths lesson to express how they are feeling. They will have to draw a face that describes how they are feeling and then write why they felt like that.

### Comment [JV1]:

2.3 Curriculum, assessment and reporting Teachers need to be able to identify students' prior knowledge in order for them to design learning sequences and lesson plans.

Teachers can identify students prior knowledge by looking at assessments from previous lessons or by looking at the curriculum to see what students learnt in the previous year.

#### Comment [JV2]:

2.3 Curriculum, assessment and reporting.

Teachers are able to see what students know by using a variety of assessment strategies. Assessment strategies that I have used in this unit include observing students, writing anecdotal notes, checklists, rubrics and providing students with open tasks.

#### References

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Wilkie, K. (2014). Upper primary school teachers' mathematical knowledge for teaching functional thinking in algebra. *Journal of Mathematics Teacher Education, 17*(5), 397-428.

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# MATHEMATICS UNIT PLANNER

Topic: Algebra- Number and growing Patterns		Year Level: 3	Term:	Week:	Date:	
Key mathematical understandings       (2-4 understandings only; written as statements believed to be true about the mathematical idea/topic):         Patterns can be described, extended, created and generalized by preforming both addition and subtraction.	Key AusVELS Focus / Content strand(s): Sub-strand(s): Level descriptions: • Describe, continue,	Standard (taken directly from Au Number and Algeb Patterns and Algebra and create number patterns	usVELS documer	nts): m performing addition c	or subtraction.	_
Patterns help develop number sense, ordering, counting and sequencing.	<ul> <li>Proficiency strand(s):</li> <li>Understanding: Uraddition and subtract</li> <li>Problem Solving: \$</li> <li>construct number and subtract</li> </ul>	Understanding Pro- nderstanding that patterns ca ction. Solving the rule for both nun nd growing patterns.	oblem Solvir an be descril nber and gro	g Reasoning bed, extended and creat wing patterns, being abl	ed by preforming both e to continue the pattern and	<b>Comment [JV3]:</b> Teachers need to have knowledge and an understanding of the content so they are able to plan meaningful learning experiences for students.
<ul> <li>Key skills to develop and practise (including strategy ways of working mathematically, language goals, etc.) (4-5 key skill only):</li> <li>Solving the rule for the number/growing pattern using addition and subtraction.</li> <li>Identifying the changes in patterns.</li> <li>Generalising patterns. Solving the next item/number in the pattern.</li> <li>Identifying the relationship between growing patterns and number patterns.</li> <li>Using appropriate language when describing number/ growing patterns.</li> </ul>	<ul> <li>Key equipment /</li> <li>Students math</li> <li>Students math</li> <li>Students math</li> <li>Interactive wh</li> <li>A3 pieces of p</li> <li>Counters.</li> <li>Large Hundre</li> <li>Hundreds cha</li> <li>Calculators</li> <li>Geometric tile</li> <li>Graph paper.</li> </ul>	<b>resources:</b> h's books. h's journals hiteboard. baper. eds chart art worksheets x 30.	a • • •	Key vocabulary (be specif ppropriate to use with students Patterns- Patterns a following a rule or rul Growing Patterns-, decreases in size by Increasing- becomin Decreasing- becomin Next- coming imme Before- in front of. Adding- to bring two together to make a n Subtracting- take an another to calculate Take Away Plus (Algebraic) Rule- is relationship between	ic and include definitions of key words re things that are arranged by les. A pattern that increases or following a rule or rules. ng bigger/greater in size. hing smaller in size. diately after the present one. o or more numbers or things ew total. way a number or amount from the difference. a method for describing the two numbers or objects.	<b>Comment [JV4]:</b> It is vital that teachers can identify skills that students will need to learn in orde for them to solve mathematical problems.
Possible misconceptions (list of misconceptions related the mathematical idea/topic that students might develop):           - Students have difficulties with describing a generalising patterns and identifying funct rules (Wilkie, 2014).           - Students find it difficult to describe pattern due to their lack of appropriate language to is needed to describe relationships and	Key probing que develop understanding probing questions):       Ind     What is       Ind     What is       Ind     What is       Ind     Can yo       Ind     How diagonal       Ind     How diagonal       Ind     How diagonal	stions (focus questions that will b to be used during the sequence of le the pattern? changing in your pattern? u describe your pattern to m d you work out the pattern? v you did this? o you know the pattern is co	e used to issons; 3 – 5 c • ne? Show rrect?	inks to other contexts urrent events, literature, etc.):	i (if applicable, e.g., inquiry unit focus,	

-	additive strat 2007). Students fin patterns and generalisatic generalisatic Cooper, 200	tegies (Warr d it difficult y find working ons more diff ons (Wilkie, 2 7).	ren and Cooper, working out growing g out explicit ficult than recursive 2014 and Warren ar	<ul> <li>Did you use</li> <li>Can you prov</li> <li>Can you use</li> <li>What would the optimized optized optized optimized optimized optimized optimized optimized op</li></ul>	counters to help you ve it to me? materials to prove it he next number be? know? Can you prov	i? t to me? ? ve it to me?	?			<b>Comment [JV5]:</b> When teaching mathematics, teachers should research or have knowledge of common misconcentions that turdents have
Learning strategies/ skills	Analys Check Classif Co-oper Considering Desigr Elabora	sing ing ying ating g options hing ating	Estimating Explaining Generalising Hypothesising Inferring Interpreting Justifying	Listening Locating information Making choices Note taking Observing Ordering events Organising	Performing Persuading Planning Predicting Presenting Providing feedback Questioning	Recog Recog Re Re Re Res Re	eading nising bias flecting porting <mark>ponding</mark> stating avising	Seeing patterns Selecting informati Self-assessing Sharing ideas Summarising Synthesising	Testing Viewing Visually representing Working independently Working to a timetable	Teachers need this knowledge so they are able to identify what students find difficult and so they can assist students by teaching them appropriate mathematical methods.
MATH F (what you to come to result of th succin	EMATICAL OCUS want the children o understand as a nis lesson – short, nct statement)	(WHOLE (a short, sha focus of the context for v independent a problem pos open-ende rea	UNING IN' E CLASS FOCUS) arp task relating to the lesson; sets the scene/ what students do in the aspect. e.g., It may be a sed, spider diagram, an ad question, game, or ading a story)	'INVESTIGATIONS SESSION' (INDEPENDENT LEARNING) (extended opportunity for students to work in pairs, small groups or individually. Time for teacher to probe children's thinking or work with a small group for part of the time and to also conduct roving conferences)	'REFLECTION & CONNECTION SESSION (WHOLE CLASS F (focused teacher quee summary to draw out the and assist children to ma This may occur at partii during a lesson. Use o strategy, gallery wa	MAKING ONS Y OCUS) stions and mathematics ake links. NB. cular points of spotlight, alk, etc.)	ADA - En: (to allow those engage in activ the in - Ext (questions t thinking)	PTATIONS abling prompt experiencing difficulty to e experiences related to itial goal task) anding prompt hat extend students' on the initial task)	ASSESSMENT STRATEGIES (should relate to objective. Includes what the teacher will listen for, observe, note or analyse; what evidence of learning will be collected and what criteria will be used to analyse the evidence)	
Sessio	n 1 scribing terns by p counting 2's, 5's and s. ing a culator to ow a rule.	Find the F the 100's of Ask studen - W yc ha cc se - W sta to to th str th	Pattern: Show chart to students. nt: /hat is a number attern? /hat pattern do bu think we would ave if we ploured in every econd number? /here would we art colouring? ive students time o discuss their inking with udents around em.	Creating patterns on hundreds chart activity & 100's chart (See Appendix 1). Students will select a rule and follow the rule by colouring the numbers on their hundreds chart. Questions: Can you describe your pattern to me? Is your pattern increasing/decreasing ?	<ul> <li>Sharing Time <ul> <li>A range of studem</li> <li>asked to describe</li> <li>patterns with the of</li> </ul> </li> <li>What number did start with? <ul> <li>What is your num</li> <li>pattern increasing</li> <li>decreasing by?</li> </ul> </li> <li>Ask students: <ul> <li>How can counting</li> <li>be helpful in every</li> </ul> </li> <li>Get students to their maths jou how they feel maths lesson Appendix 2).</li> </ul>	ts will be their class. d you hber or by 2's rday life? to write in urnals on after the (See	Enabling ( - Wi pa - Wi ab - Wi diff the se the - Ca the - Ca the yo cre pa inc wh	prompts: nat is a number ttern? nat do you notice out this pattern? nat is the ference between a first and cond number in a pattern? in you describe a pattern to me? rou started at mber two and u wanted to pate a number ttern that creases by two, nat would the	Teacher will take anecdotal notes on children (See Appendix 3). The teacher will rove around the classroom asking students questions to help them know what the student is thinking and their understanding of the task. Teacher will reflect and write notes on the following: • Were students able to skip count on the calculators? If not, what was	<ul> <li>Comment [JV7]: 2.5 numeracy strategies: Allowing students to use resources to support their learning is very beneficial. This is particularly beneficial for visual learners, as they are able to visualise the mathematical problem.</li> <li>Using the hundreds chart helps students identify patterns.</li> <li>Comment [JV6]: 2.2 Content selection and organisation.</li> <li>Teachers need to organise content into an effective learning and teaching sequence.</li> <li>If teachers have knowledge and an understanding of the content, they would be able to plan an effective learning sequence.</li> </ul>

	Skip Count with the 100's chart: Students are going to skip count by 2's, 5's and 10's referring to a 100s chart. To make it more challenging ask children to skip count backwards. Calculator Count: Students will sit in a circle with their calculators. Show students how to skip count forwards by 1's and then backwards with their calculators. As a class, skip count by 10's, 5's and 2's.			<ul> <li>next number be? Continue the pattern using your calculator.</li> <li>Extending prompts: <ul> <li>If you started at 108 and you were counting by 3's, what would be the next number? Continue the number pattern.</li> </ul> </li> <li>If you started at 108 and you were counting down by 3's what would be the next number? Continue the pattern.</li> </ul>	<ul> <li>confusing?</li> <li>Were students able to describe how numbers changed in a skip counting pattern?</li> <li>Which students were able to skip count without calculators?</li> <li>Did I help students use what they know about patterns to recognise their own mistakes in hundred chart patterns?</li> </ul>
Session 2  Describing patterns using addition and subtraction.	The teacher will give each student a multiple of two. The students have to put the numbers in order from smallest to largest without speaking. <b>Questions:</b> What is the number pattern? How do you know? Can you explain your thinking to the class?	Describe the number pattern activity (See Appendix 4). • Teacher will go around to each pair, ask them questions and observe their mathematical thinking. Questions - Can you describe this number pattern to me? - What is the rule? - How do you know?	<ul> <li>Reflection time.</li> <li>What did you learn today?</li> <li>How did you work out the number patterns?</li> <li>(Allow students time to describe their strategies-students can learn from one another).</li> <li>Get students to write in their maths journals on how they feel after the lesson (See Appendix 2).</li> </ul>	<ul> <li>Enabling prompts:</li> <li>What is the difference between the first and second number in the number pattern?</li> <li>How did you work out the difference between the first and second number?</li> <li>Can you see any changes in the pattern? What are they?</li> <li>Can you use counters to show</li> </ul>	<ul> <li>Assessment will be taken in a checklist for this lesson. (See Appendix 5).</li> <li>Where there any students that needed teacher assistance?</li> <li>Which students used counters to assist them?</li> <li>Were students using addition and subtraction to work out the number patterns?</li> <li>Which students were</li> </ul>

		- Can you prove it by using counters?		me the pattern? Extending prompts: - How do you know this number pattern is correct? Can you prove it? - What will come next in this pattern? - How do you know that is correct? Prove it to me using counters.	<ul> <li>able to explain the number patterns verbally?</li> <li>Which students were able to write their description clearly in their maths books?</li> </ul>	
Session 3 • Continuing the number pattern and growing pattern.	Continuing the number pattern: As a class work out the next few numbers in the number pattern using the scootle website that is below. Allow students to come up and click on the numbers. Complete five –six number patterns with students. Scootle website: http://www.scootle.edu.au/ ec/viewing/L6551/asset1.ht ml Questions: -What is the number pattern? -What strategies did you use to work it out? -What is the next number in the pattern?	<ul> <li>Think, Pair, Share</li> <li>The teacher will put on the interactive whiteboard three growing patterns (See Appendix 7).</li> <li>Growing pattern activity (See Appendix 8).</li> </ul>	Reflection time. Ask students to sit in a circle. The teacher will get a soft volleyball. When the student receives the ball they have to answer one of the following questions: -What is something you learnt/enjoyed from the lesson? -What strategies did you use? -What strategies did you use? -What is something you felt challenging and why? When a student receives the ball they have to answer a question then they have to roll the ball to another student.	<ul> <li>Enabling prompts: <ul> <li>Can you describe your pattern to me?</li> <li>Can you see any changes in the pattern? What are the changes you see?</li> <li>What number is your pattern increasing/decreas ing by?</li> </ul> </li> <li>Extending prompts: <ul> <li>What will come next in this pattern? How do you know?</li> <li>Can you use counters to prove this to me?</li> </ul> </li> </ul>	Assessment was taken through anecdotal notes for this lesson (See Appendix 3). - Where the students able to identify the growing pattern? - Where the students able to use addition and subtraction to work out their growing pattern? - Which students needed further assistance? - Where the students able to generalise the seventh pattern? Did they have to use counters to assist them?	Comment [JV8]: 2.6 Information and Communication Technology. Technology plays a big part in both teaching and learning. Using technology in the classroom is both interactive and engaging. The important role that technology plays in education gives teachers the opportunity to design meaningful learning experiences that embed technology. One technological device that teachers can use is the interactive whiteboard.

Section 4	<ul> <li>Growing patterns Teacher will put a growing pattern on the interactive whiteboard (See Appendix 6).</li> <li>Questions: -What is a growing pattern?</li> <li>What will come next in this growing pattern?</li> <li>Allow students to have a discussion with the students around them.</li> <li>As a class draw the next three objects and write the number sequence.</li> </ul>		lesson (See Appendix 2).		Studente will be appeared	
<ul> <li>Student will be creating their own growing patterns by using addition and subtraction.</li> </ul>	The students will all sit in a circle. The teacher will tell the students that today we are going to be creating our own growing patterns. Teacher will ask a student to grab a small handful of counters. Whatever the amount is will be our rule we have to follow. Students can choose if they want to create an increasing or decreasing pattern. *Repeat this process three times.	<ul> <li>Students will create their own growing pattern using counters. (Make sure students leave the pattern on their tables).</li> <li>Students will then draw their growing pattern into their books and identify the rule.</li> <li>Students will then find a partner, their partner needs to draw their growing pattern in their maths book, they need to identify the rule and then continue the pattern three more</li> </ul>	<ul> <li>Gailery Walk Students display their patterns at their tables and students walk around the classroom observing the different patterns their peers have made.</li> <li>Questions: <ul> <li>What strategies did you use to create your growing patterns?</li> <li>What is something you learnt from the lesson?</li> </ul> </li> <li>Give students an opportunity to write in their maths journals on</li> </ul>	<ul> <li>What is the difference between the first and second object?</li> <li>What is changing in the growing pattern?</li> <li>Extending prompts: <ul> <li>What would the tenth object look like in the growing pattern?</li> <li>Can you prove it to me using counters?</li> </ul> </li> </ul>	<ul> <li>Students will be assessed through a checklist (See Appendix 9).</li> <li>Where students able to create their own growing pattern?</li> <li>Where students able to identify their partners growing pattern?</li> <li>Where they able to continue the growing pattern?</li> <li>Where students able to write the number sequence?</li> </ul>	Comment [JV9]: 2.5 numeracy strategies: Using concrete material to support students learning.

	Ask students: -Are we doing to create an increasing/decreasing pattern? -What number did you want to start with? -What would come next in our growing/decreasing pattern? -How did you work it out?	times. *Teacher will rove around the classroom questioning students. -What is the growing pattern? -Is it increasing or decreasing? -What will the fifth object look like? -Can you prove it to me using counters?	how they feel about the lesson (See Appendix 2).			
Session 5 Creating growing patterns. Revising number/gr owing patterns.	The teacher will ask students to get into groups of five. The teacher will tell students a rule. Students have to work together to create a growing pattern following the rule. Students will have to describe their pattern with the class. <b>Questions the teacher</b> <b>will ask.</b> - Can you describe your growing pattern to the class? - What is the number sequence to your growing pattern? - What would the next object look like?	Open Task Students will be creating both growing and decreasing patterns using geometric tiles. (See Appendix 10).	Reflection/Revision         The students will be asked to sit in a circle.         The teacher will put an A3 piece of paper in the middle and ask students to write anything they learnt/ know about number patterns and growing patterns.         Allow students time to have a discussion with the students around them.         Teacher will prompt students:         -What is a number pattern?         -What strategies did we learn?	<ul> <li>Enabling prompts:         <ul> <li>What is the difference between the first number and second number?</li> <li>What is changing in your pattern?</li> </ul> </li> <li>Extending prompts:         <ul> <li>What would the 8<sup>th</sup>, 9<sup>th</sup>, 12<sup>th</sup> object look like in your growing pattern?</li> <li>How do you know?</li> <li>Can you prove it ?</li> <li>What is the rule for your growing pattern?</li> </ul> </li> </ul>	Assessment with be through a rubric based on the open task (See Appendix 11). - Where students able to create a range of growing tasks? - How many were students able to create? - What strategies did students use to create their growing patterns? - Did they use addition, subtraction, halving, doubling? - Who needed further assistance in helping them create a growing pattern?	<ul> <li>Comment [JV10]: 2.5 numeracy strategies.</li> <li>Teachers can provide students with open-ended tasks as they promote problem solving.</li> <li>Open tasks are effective as they enable the whole class to participate, they can easily be modified or extended in order to scaffold learning. Open tasks promote engagement in tasks that will enable our students to reason effectively, use systems thinking, make judgments and decisions, and solve problems.</li> <li>Comment [JV11]: 2.5 numeracy strategies: Using concrete material to support student learning and to cater for students.</li> <li>Concrete material is beneficial for visual learners.</li> </ul>

## Literature review

## Algebra

Algebra is a mathematical content area that is explored in both primary and secondary school settings. Literature states that algebra is a study of relationships and patterns. It is a way of thinking as it enables you to analyse models/representations, generalise and justify (Reys et al, 2013; Miller and Warren, 2012 and Wilkie 2014). Algebra can be seen as an art as it has order and consistency, it is also a language that uses symbols and terms (Reys et al, 2013 and Stacey & MacGregor, 1997).

## **Thinking Process in Algebra**

Lannin (2005), Miller and Warren (2012), Warren and Cooper (2007) and Wilkie (2014) indicate that generalisation is the main concept that students must know in order to work out algebraic problems. Lannin (2005); Warren and Cooper (2007) and Wilkie (2014) argue that generalisation needs to take place in order for students to understand algebra. For students to understand generalisations they must know that an algebraic rule is a generalisation (Wilkie, 2014). Wilkie (2014) goes deeper and states that there are two types of generalisations that students need to know to think functionally, these are recursive and explicit generalisations. A recursive generalisation is the relationship between successive items in a pattern, whereas explicit generalisation is the relationship between successive items in a pattern, whereas explicit generalisation is the relationship between successive items in a pattern, whereas explicit generalisation is the relationship between successive items in a pattern, whereas explicit generalisation is the relationship between two variables/quantities (Wilkie, 2014). Brown (2008) argues that reversibility, generalisation and flexibility are crucial aspects of mathematical thinking and are needed when investigating or problem solving. Warren, Mollinson and Oestrich (2009) states that it is students thinking process that enable them to understand algebra. Algebraic thinking looks at the process, rather than just looking at the final product (Stacey & MacGregor, 1997 & Warren, Mollinson and Oestrich, 2009). Warren, Benson & Green (2007) and Wilkie (2014) both agree that functional thinking is also a central focus of algebra, as it enables students to look for relationships between two quantities.

## Key understandings

A key aspect of functional thinking is being able to identify a rule that shows how two variables are related (Wilkie, 2014). Warren, Benson & Green (2007) and Stacey & Macgregor (1997) state that functional thinking helps students identify which operations are related to each other. Hunter (2015) points out that students need to understand and have knowledge of the communicative, associative and distributive principles to be able to solve equations and reason algebraically. Knowledge of the equals sign and equivalence is also an important concept students must know (Falkner, Levi and Carpenter, 1999; Hunter, 2012 and Warren, Mollinson, & Oestrich, 2009). Students must understand that the equal sign is a mathematical symbol used to express equality and balance (Falkner, Levi and Carpenter , 1999 and Warren, Mollinson, & Oestrich, 2009). Warren, Mollinson and Oestrich (2009) states that in order for students to understand equivalence, students need to develop language that helps describe both equivalent and non-equivalent situations. Eg: "same as" "different from" "not equal". Students need to understand that equality means that two expressions are equal, meaning they have the same value. Teachers need to write equations in a range of ways so that students are able to develop a meaning of equivalence (Warren, Mollinson and Oestrich, 2009). Analysing and generalising repeating and growing patterns is another concept that is explored in algebra (Warren and Copper, 2006). Warren and Cooper, 2006 and Wilkie 2014, state that investigating these two types of patterns is a powerful way to help students explore functional thinking (Warren and Cooper, 2006 and Wilkie, 2014). Another concept that is explored in algebra are variables. Variables are a symbol for an unknown number, letters are usually used to represent the unknown (Stacey & MacGregor, 1997).

## Appropriate Teaching Practices:

There are many teaching approaches to develop student's algebraic thinking. First, it is important that algebra is not taught in isolation, it should be integrated into everyday maths lessons (Hunter, 2012 and Hunter, 2015). Hunter (2015) and Warren, Mollinson and Oestrich (2009) state that algebra and arithmetic must go together, therefore teachers need to plan tasks that link them together. Reys et al (2013) indicates that we can help students develop algebraic

**Comment [JV12]:** 2.1 content and teaching strategies of the teaching area: It is essential that teachers research the content before teaching it to students. Teachers need to have a broad understanding of the content so they are able to develop engaging activities. thinking by modelling problems, patterns and relations. Teachers should provide opportunities for generalising by extending tasks and plan activities that expand students understanding of the role of the equals sign (Hunter, 2012). In order to cater for the different learning styles students' should be exposed to concrete material and a range of representations (Rivera& Becker, 2005; Stacey and Macgregor, 1997; Warren, Benson, Green, 2007; Warren & Cooper, 2007; Warren, Mollinson, Oestrich, 2009 and Wilkie, 2014). Wilkie (2014) further argues that students should be given tasks that are meaningful and have a realistic context for students to understand the purpose of algebra. Wilkie (2014) states that students can learn to recognise a function by verbally describing their patterns and then expressing them algebraically. Brown (2008) and Hunter (2015) both support Wilkie (2014) as they believe students should be given the opportunity to discuss and justify their thinking with others. This is very beneficial as it develops new thinking, clarifications, modifications and it identifies misconceptions (Brown, 2008). Stacey and MacGregor (1997) point out that students should discuss the different methods they used as students can learn from one another. Hunter (2015); Reys et al, (2013) and Wilkie (2014), state that questioning students is an important strategy teachers can use when helping students develop algebraic thinking. When we ask students questions they have an opportunity to share, discuss and justify their mathematical thinking and understanding. This can help teachers assess what students know.

# **Comment [JV13]:** 2.5 Numeracy teaching strategies:

Teachers should allow students to use concrete material as it caters for a range of different learning styles.

Concrete material aids students' understanding and helps students to construct meaning.

**Comment [JV14]:** 2.5 Numeracy teaching strategies.

Teachers need to provide opportunities for students to have group discussions and work with one another. Students are able to share ideas, identify any misconceptions and most importantly learn from one another.

# APPENDICES

# APPENDIX 1: Creating patterns on hundreds chart activity.

- Students need to get a hundreds sheet.
- The students are going to create colourful patterns by following a rule.
- After they have selected a rule, the students should colour each square with the numbers that follow the rule.
- Students will use their calculators to assist them.
- Students can start at any number they like.
- Students can create as many patterns as they like. They must use different colours for each rule.
- 1. Increasing **or** decreasing by two.
- 2. Increasing or decreasing by three.
- 3. Increasing **or** decreasing by four.
- 4. Increasing **or** decreasing by five.

Hundred-Board Wonders (Cuevas and Yeatts, 2005).

(E		10	20	30	40	50	60	70	80	90	100
	300	6	19	29	39	49	59	69	79	89	66
		80	18	28	38	48	58	89	78	88	86
	K-L	7	17	27	37	47	57	67	77	87	79
Chart		9	16	26	36	46	56	99	76	98	96
00		5	15	25	35	45	55	65	75	85	95
Ć	)	4	14	24	34	44	54	64	74	84	94
		3	13	23	33	43	53	63	73	83	93
		2	12	22	32	42	52	62	72	82	92
		-	11	21	31	41	51	61	71	81	16

Name:

# APPENDIX 2: Students maths journal.

- Draw a face that describe how you felt today after your maths lesson.
  - (Is it a happy face, a sad face, a confused face, a crying face?)
- I feel like this because......
- Next lesson I can.....

# **APPENDIX 3: Anecdotal notes.**

Students Name / Date:	Notes (Difficulties/misconceptions)	Action required	Action taken: (When, How)

**APPENDIX 4: Describe the number pattern activity.** 

- Students will work in pairs for this task.
- Each pair will be given the following five number patterns and counters to assist them.
  - 1. 8,12,16,20,24,28,30.
  - 2. 28, 25, 22, 19, 16, 13.
  - 3. 19, 24, 28, 34, 39, 44.
  - 4. 10, 20, 40, 80, 160, 320
  - 5. 61, 59,58, 55, 53, 51
- Students have to work together to identify if the number patterns are correct or not. If the number pattern is correct they have to describe the number pattern. They then have to write the number pattern and description in their maths books.
- If the pattern is incorrect students have to work out what the pattern is and identify which number is incorrect in the pattern.
- Students then need to write the correct number pattern in their books and write their description in their maths book.

# **APPENDIX 5: Checklist**

Students Name				
Used counters or other material to assist them.				
Used addition and subtraction to solve the number patterns.				
Ability to describe the pattern verbally.				
Ability to describe the number pattern in writing.				
Needed further assistance from the teacher.				

**APPENDIX 6: Growing Pattern** 

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**APPENDIX 7:** Growing patterns for students.









# **APPENDIX 8: Growing pattern activity explanation.**

Students have to...

- Choose one growing pattern.
   Draw the growing pattern in their book.
- 3. Work out the rule.

- 4. Find the fifth object in the pattern.
   5. Find the seventh object in the pattern.
   6. Write the number pattern sequence of the object pattern.
- The teacher will write the steps on the whiteboard for students to follow.
- Counters will be on each table for students to use if they need to. ٠

(National Council of Teachers of Mathematics, 2015).

# **APPENDIX 9: Checklist**

Students Name:				
Student created a growing pattern using counters.				
Student described the rule of their growing pattern.				
Student continued their partners growing pattern successfully.				
Student identifies their partners rule to their growing pattern.				
Needed further assistance.				

# APPENDIX 10: Open Task- Growing Patterns.

## **Growing Patterns**

Directions:

Using graph paper and geometric tiles, make a series of growing patterns. You can make as many growing patterns as you like. Explain your growing pattern rule.

Fill out the following chart for you **best** two growing patterns on this chart.

Growing Pattern A: Draw the first three figures of Growing Pattern A in the boxes below.							
Explain the rule for Growing Pattern A:							
Write the number sequence for Pattern A:							

Growing Pattern B: Draw the first three figures of Growing Pattern B in the boxes below.								
Explain the rule for Growing Pattern B:								
Write the number sequence for Pattern B:								

# APPENDIX 11:

# Growing Patterns Open Task- Rubric

Goes beyond	Student is able to create six or more growing patterns by using the geometric tiles. They are able to choose two of their favourite growing patterns, and draw the first three figures of their pattern on the template given to them.
	Students demonstrate a clear understanding of the task by using one or more efficient mathematical strategies that is suitable for the task. (Example: uses addition and subtraction, doubling, halving).
	The student uses appropriate words to describe the rule and is able to write the number sequence for both growing patterns.
Task Accomplished	Student was able to create 5-6 growing patterns using the geometric tiles. The student was able to choose two of their favourite growing patterns, and draw the first three figures of their pattern on the template given to them.
	Student was able to display a clear understanding of the task by using one correct mathematical strategy such as addition, subtraction, halving, doubling.
	The student uses appropriate words to describe the rule and is able to write the number sequence for both growing patterns.
Substantial Progress	Student was able to create 3-5 growing patterns using the geometric tiles. They are able to choose two of their favourite growing patterns, and draw the first three figures of their pattern on the template given to them.
	Student was able to use a suitable strategy to create their growing patterns, such as adding, subtracting, doubling, halving.
	Student was not clear when explaining the rule of the growing pattern they had created and was unable to identity the number sequence.
Some Progress	Student was able to attempt the task by creating two growing patterns using the geometric tiles. They are able to draw the first three figures of both patterns on the template given to them.
	The student used a suitable strategy for creating their growing patterns such as adding, subtracting, doubling and halving.
	The student was unable to communicate the rules or the number sequences of both growing patterns.
Little Progress	Student made little or no evidence of engagement in the task.
	Student was able to complete one growing pattern. The student did not explain the rule of the growing pattern and did not write the number sequence for their pattern.